Transverse cracking and Poisson's ratio reduction in cross-ply carbon fibre-reinforced polymers

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Gradual damage development in carbon fibre-reinforced polymers (CFRP) and its effect on the mechanical properties have been important subjects of investigation for many years. Most authors have studied transverse matrix cracking in cross-ply lay-ups and used the longitudinal Young's modulus as an indicator of the extent of damage development. Reductions of typically only a few percent have been found at saturation crack spacing. Some authors have studied the effect of matrix cracking on Poisson's ratio. The results show large reductions, but few data are available on the evolution of Poisson's ratio throughout the process of gradual matrix cracking and on the influence of the 0°/90° ply thickness ratio. Moreover, none of the available models seems to accurately predict the quantitative evolution of Poisson's ratio. In this work the degradation of the longitudinal and the transverse properties of a number of cross-ply CFRP laminates due to transverse matrix cracking under longitudinal tension was studied. The longitudinal Young's modulus appeared to be less sensitive to damage development, in contrast to Poisson's ratio which exhibited significant reductions in all lay-ups. A micromechanical model, based on the shear lag theory, was developed to predict the evolution of Poisson's ratio and the effect of the 0°/90° ply thickness ratio. The correlation between experiment and theory was very satisfactory.

1. Introduction

Laminated fibre reinforced polymers have been under study for many years due to their high strength and stiffness over weight ratio and the fact that the directionality of their mechanical properties can be tailored to meet the requirements of any given application. One of the key features of this material class is their damage initiation and propagation behaviour which, in contrast to monolithic materials, is spatially distributed in nature and comprises a variety of mutually interacting damage modes. The most common damage modes are matrix cracking, delamination growth and fibre fracture.

The initiation and propagation of each of these damage modes in a wide variety of lay-ups and their influence on the mechanical properties for different loading conditions has been a subject of investigation for many years. Attention has been mainly paid to the phenomenon of transverse matrix cracking in cross-ply laminates. A review of the available literature results shows that the damage parameter under investigation has always been the longitudinal Young's modulus (it should be noted here that longitudinal in this case refers to the loading direction). Significant reduction of this parameter, accounting for the stiffness reduction in the longitudinal direction, has only been found when either glass fibres were used [1–3] or the 0°/90° ply thickness ratio was small [2–5]. The use of stiff carbon fibres strongly reduces the influence of transverse matrix cracking on the Young's modulus [6, 7].

The evolution of Poisson's ratio in the presence of transverse matrix cracking has been studied by some authors [1, 6–10]. This transverse property appeared to exhibit large reductions and is very sensitive to the phenomenon of matrix cracking, both in GFRP and CFRP laminates. Most authors have, however, only reported degradations at saturation crack spacing or have only considered one type of lay-up. A number of models are available in the literature which attempt to theoretically predict the evolution of Poisson's ratio as a function of matrix crack density. Shear lag theory [6–9], elasticity analysis [11] and continuum damage mechanics [7, 10] have all been used as modelling approaches. Only the model of Nuismer and Tan [8, 9], which is based on a modified shear lag analysis, does appear to predict both the qualitative and the quantitative nature of the Poisson's ratio degradation. The major drawback of this model, is that it requires lamina stiffness properties as input which are hard to determine experimentally.

In this work a systematic study will be made of the influence of transverse matrix cracking on the evolution of both the longitudinal Young's modulus and the Poisson's ratio in a variety of cross-ply lay-ups. The lay-ups differ in their 0°/90° ply thickness ratio. Both shear lag theory and concepts developed for the modelling of

TABLE I Basic mechanical properties of cross-ply and unidirectional laminates

	$E_{x,\,\text{init}}$ (GPa)	$V_{xy,\,\text{init}}$	σ_{max} (MPa)	t_0 (mm)	t_{90} (mm)
$[0, 90_3]_s$	35	0.029	540	0.125	0.375
$[0_2, 90_2]_s$	63	0.070	1160	0.250	0.250
$[0_3, 90]_s$	95	0.097	1590	0.375	0.125
$[0_8]$	131	0.620			
$[90_8]$	9	0.027			

transverse strain behaviour in ceramic matrix composites will be used in an attempt to theoretically predict the Poisson's ratio degradation.

2. Material and experimental procedure

All materials used in this study were produced starting from a Vicotex 6376/35/137/T400H prepreg from Ciba Geigy, consisting of the Vicotex 6376 high toughness resin and Torayca T400H high strain carbon fibres. Three different lay-ups were produced having different 0°/90° ply thickness ratios: $[0, 90_3]_s$, $[0_2, 90_2]_s$ and $[0_3, 90]_s$. The main goal of using different lay-ups was to quantitatively study the effect of the 0°/90° ply thickness ratio on the degradation behaviour. The 0°-direction corresponds to the loading direction used in the tensile tests and will also be indicated with the subscript x, whereas the 90°-direction will be indicated as y.

The basic mechanical properties (longitudinal Young's modulus $E_{x,\,\text{init}}$, ultimate tensile strength σ_{max}, major Poisson's ratio $\nu_{xy,\,\text{init}}$) of the different lay-ups in the undamaged state were determined by performing separate tensile tests and are presented in Table I. This table also contains the stiffness properties of both a $[0_8]$ and a $[90_8]$ laminate and the 0° and 90° half ply thicknesses t_0 and t_{90}, since they will be needed for modelling purposes.

To investigate the degradation of both the longitudinal Young's modulus E_x and Poisson's ratio ν_{xy} during interrupted cyclic tensile loading, tensile samples (length 150 mm, width 12 mm, thickness 1 mm) were produced to which a 0/90 strain gauge (type FCA-6-11, Tokyo Sokki Kenkyujo Co, 6 mm gauge length) was attached. All samples were mounted in a MTS 810 servohydraulic testing machine and tested in interrupted tension. Stress was raised linearly up to an initial maximum stress after which the samples were partially unloaded. Subsequently a replica was taken of a polished edge and unloading was completed. This procedure was repeated and during each cycle the maximum stress level was increased. The tests were stopped when the maximum stress level was just below the previously determined ultimate tensile strength.

The stress values used depended on the lay-up and were determined based on the ultimate tensile strength. Maximum stress values between 225 and 500 MPa were used for the $[0, 90_3]_s$ lay-up, between 400 and 1000 MPa for the $[0_2, 90_2]_s$ lay-up and between 900 and 1500 MPa for the $[0_3, 90]_s$ lay-up.

The replication technique was used to determine the number of transverse matrix cracks present in the specimens at each maximum stress level. A zone of 40 mm around the location of the strain gauge was investigated in each case. By combining the strain gauge and replica results, curves could be obtained of both the longitudinal Young's modulus and the Poisson's ratio as a function of matrix crack density.

The replication technique was quite cumbersome and the crack density evolution of only one specimen per lay-up was determined using this technique. Mechanical testing was more straightforward and the evolution of the Young's modulus and the Poisson's ratio were determined for several specimens of each lay-up. Mechanical test results proved to be very reproducible.

3. Results and discussion

3.1. Matrix crack evolution

Fig. 1 shows the evolution of the transverse crack density as a function of the applied maximum stress level for all cross-ply lay-ups. Transverse crack density ρ_{90} was obtained by dividing the observed number of matrix cracks by the length of the investigated zone.

The stress at which cracking initiates increases as the 0°/90° ply thickness ratio increases: 300 MPa for $[0, 90_3]_s$, 600 MPa for $[0_2, 90_2]_s$ and 1000 MPa for $[0_3, 90]_s$. This is caused by the increasing constraint exerted by the 0°-layers and is in agreement with literature results. After initiation, the matrix crack density increases steadily with increasing maximum tensile stress. In all cases a saturation crack density is reached near the ultimate tensile stress. The saturation crack density increases as the 0°/90° ply thickness increases: 0.8 cracks/mm for $[0, 90_3]_s$, 1.2 cracks/mm for $[0_2, 90_2]_s$ and 2.3 cracks/mm for $[0_3, 90]_s$. This is in agreement with the shear lag theory.

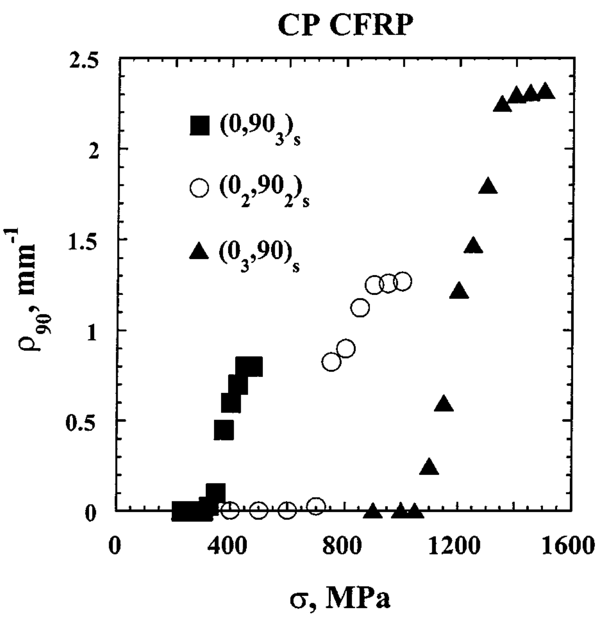

Figure 1 Matrix crack density ρ_{90} as a function of stress for all lay-ups under investigation.

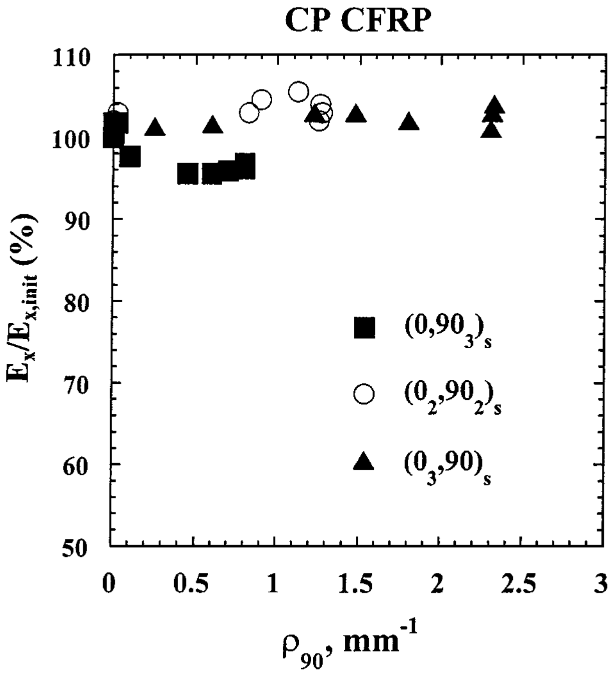

Figure 2 Longitudinal Young's modulus degradation as a function of matrix crack density ρ_{90} for all lay-ups under investigation.

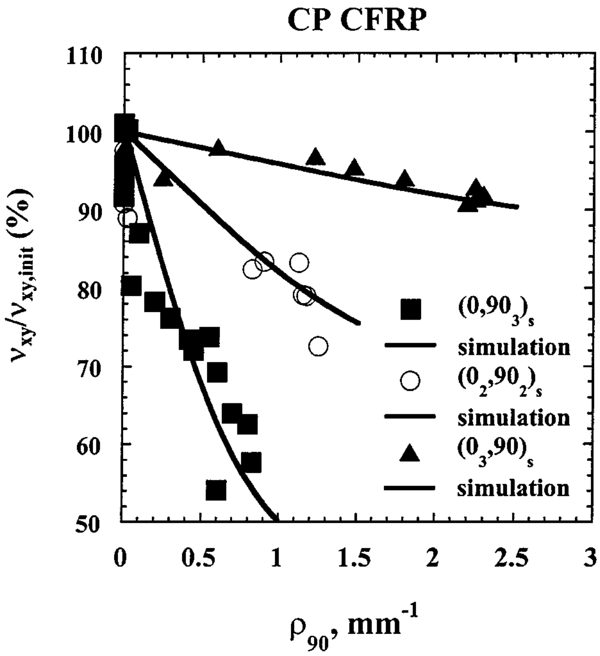

Figure 3 Poisson's ratio degradation as a function of matrix crack density ρ_{90} for all lay-ups under investigation (experimental results and theoretical prediction).

3.2. Degradation of the longitudinal Young's modulus

Fig. 2 shows the evolution of the longitudinal Young's modulus as a function of matrix crack density. Young's modulus is displayed on the graph as a percentage of the modulus $E_{x,\text{init}}$ in the undamaged state. As can be seen, the longitudinal Young's modulus exhibits little or no degradation as the matrix crack density increases. A clear downward trend in the data can only be observed for the $[0, 90_3]_s$ lay-up and even for this low 0°/90° ply thickness ratio a maximum degradation of only 4% is observed. The other lay-ups do not show any measurable degradation and a slight increase can even be noticed in these cases which is probably due to a better alignment of the fibres after initial loading.

One can conclude from the results presented in Fig. 2 that the longitudinal Young's modulus does not reveal the amount of transverse cracking because this damage mode has little or no influence on the Young's modulus. This is in agreement with literature results and is due to the high stiffness of the fibres as compared to the matrix in this CFRP material. This results is most of the load being carried by the 0°-plies, even in lay-ups with a low 0°/90° ply thickness ratio. Since only small loads are carried by the 90°-plies, matrix cracking will increase the load on the 0°-plies by only a very small amount, resulting in little or no effect on the longitudinal stiffness properties.

3.3. Degradation of Poisson's ratio

Fig. 3 shows the evolution of the Poisson's ratio as a function of the matrix crack density. Poisson's ratio is displayed as a percentage of the value $\nu_{xy,\text{init}}$ in the undamaged state.

As can be seen, a clear downward trend is noticeable for both the $[0, 90_3]_s$ and the $[0_2, 90_2]_s$ lay-up and to a lesser extent for the $[0_3, 90]_s$ lay-up. The maximum degradations observed are 43% for the $[0, 90_3]_s$ lay-up, 17.5% for the $[0_2, 90_2]_s$ lay-up and 4.5% for the $[0_3, 90]_s$ lay-up. All lay-ups show a gradual degradation as the matrix crack density increases.

The degradations in Poisson's ratio are much higher than the ones observed for the longitudinal Young's modulus. This can be explained by the fact that the Poisson's ratio is a transverse property as compared to the longitudinal nature of the Young's modulus.

As illustrated by the shear lag theory, matrix crack growth in the transverse plies can significantly alter the longitudinal stress and strain distribution in these plies. However, though the relative changes in longitudinal stress and strain state can be quite large in the 90°-plies, they will have very little effect on the overall longitudinal properties of the laminate, since its behaviour in this direction is largely dominated by the stiff 0°-plies.

Any effect on the longitudinal strain behaviour of the 90°-plies will always be accompanied by a change in the transverse strain behaviour. These changes can have a large effect on the overall transverse strain behaviour of the laminate since the transverse behaviour is largely dominated by the 90°-plies. When cracking of the 90°-plies occurs, these plies carry virtually no load in the x-direction (parallel to the 0°-fibres) and therefore do not wish to contract transversely. However, because the 0° and 90°-plies are bonded together, their strain must be the same in the transverse or y direction and therefore matrix cracking reduces the average transverse strain and the Poisson's ratio of the laminate.

The validity of this explanation is illustrated in Fig. 3. Since the effects described above will be larger as the 0°/90° ply thickness ratio decreases, it is normal that larger degradations are observed when a larger amount of transverse plies is present.

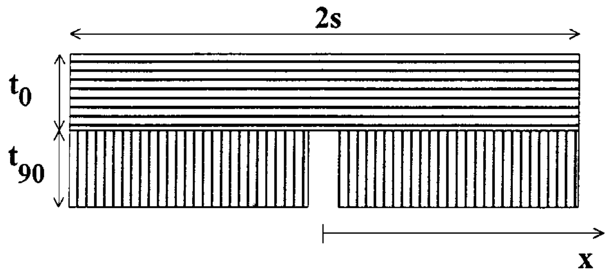

Figure 4 Schematic representation of a unit-cell with a cracked 90°-ply.

4. Modelling of Poisson's ratio degradation

4.1. Model description

The objective of the model presented in this publication is to quantitatively predict the evolution of Poisson's ratio as a function of matrix crack density in cross-ply laminates and therefore needs the number of matrix cracks per unit length measured with the replication technique. The model is based on both the shear lag theory and the principles of transverse strain modelling that have been developed by Vanswijgenhoven *et al.* [12, 13] for ceramic matrix composites.

A cracked cross-ply composite laminate is divided in unit-cells, each containing one matrix crack. Fig. 4 shows a schematic representation of a unit-cell. Only half of the composite thickness is shown, due to symmetry reasons.

The length of the unit cell 2s is determined by the matrix crack density ρ_{90}: $2\text{s} = 1/\rho_{90}$. t_0 and t_{90} are half the thickness of respectively the 0° and the 90°-ply. Other important material parameters are the longitudinal Young's modulus of the overall laminate (E_x), the longitudinal Young's modulus of the 0° and the 90°-ply ($E_{x,0}$, $E_{x,90}$), the Poisson's ratio of the 0° and the 90°-ply ($\nu_{xy,0}$, $\nu_{xy,90}$) and the shear modulus of the composite G_{t}.

The formation of a transverse crack in the 90°-ply leads to a redistribution of the stresses in the vicinity of the crack. Based on shear lag theory [6], the following expression can be obtained for the longitudinal stress in the 90°-ply $\sigma_{90,\text{L}}(x)$:

$$\sigma_{90,\text{L}}(x) = \sigma\frac{E_{x,90}}{E_x}\left(1 - \frac{\cosh(\lambda x)}{\cosh(\lambda s)}\right) \tag{1}$$

σ is the stress applied to the composite, x is the distance from the transverse crack as indicated in Fig. 4. λ is called the stress redistribution factor and is given by [6]:

$$\lambda = \sqrt{\frac{\beta G_t(t_0 + t_{90})E_x}{t_{90}^2 t_0 E_{x,0} E_{x,90}}} \tag{2}$$

β is a shear lag parameter. Its value is 1 for a linear variation of the longitudinal displacement across the transverse ply. A value of 3 would imply a parabolic variation of the displacement.

Any change in the longitudinal stress behaviour of the 90°-ply implies a change in the longitudinal stress behaviour of the 0°-ply. The longitudinal stress in the 0°-ply $\sigma_{0,\text{L}}$ is given by [6]:

$$\sigma_{0,\text{L}}(x) = \sigma\frac{E_{x,0}}{E_x} + \frac{t_{90}}{t_0}\sigma\frac{E_{x,90}}{E_x}\left(1 - \frac{\cosh(\lambda x)}{\cosh(\lambda s)}\right) \tag{3}$$

The longitudinal strain in both plies can be calculated from the elastic stress response. The longitudinal strain of the 90°-ply $\varepsilon_{90,\text{L}}$ is given by:

$$\varepsilon_{90,\text{L}} = \frac{\int_0^{\text{s}}(\sigma_{90,\text{L}}(x)/E_{x,90})dx}{\text{s}} = \sigma\frac{1 - (\tanh(\lambda s)/\lambda s)}{E_x} \tag{4}$$

The longitudinal strain of the 0°-ply $\varepsilon_{0,\text{L}}$ is given by:

$$\varepsilon_{0,\text{L}} = \frac{\int_0^{\text{s}}(\sigma_{0,\text{L}}(x)/E_{x,0})dx}{\text{s}} = \frac{\sigma}{\text{E}} + \frac{(t_{90}/t_0)\sigma(E_{x,90}/E_x)(1 - (\tanh(\lambda s)/\lambda s))}{E_{x,0}} \tag{5}$$

Using the formulas for the longitudinal strain of both plies, the transverse strains can be calculated by making use of the Poisson's ratios. The transverse strain of the 90°-ply $\varepsilon_{90,\text{T}}$ is given by:

$$\varepsilon_{90,\text{T}} = -\upsilon_{xy,90}\varepsilon_{90,\text{L}} \tag{6}$$

The transverse strain of the 0°-ply $\varepsilon_{0,\text{T}}$ is given by:

$$\varepsilon_{0,\text{T}} = -\upsilon_{xy,0}\varepsilon_{0,\text{L}} \tag{7}$$

The transverse strains as they were calculated here do not take into account the constraints both plies impose on each other. They are the strains as they would appear in a free 0° and 90°-ply under the longitudinal stress behaviour given by Equations 1 and 3. Since both plies remain in contact with each other, their transverse strain should be equal and equal to the overall transverse strain ε_{T}. This condition can be satisfied by applying transverse stresses $\sigma_{0,\text{T}}$ and $\sigma_{90,\text{T}}$ to the plies. The transverse stresses should be in equilibrium so the following equations can be written:

$$\varepsilon_{\text{T}} = \varepsilon_{0,\text{T}} + \frac{\sigma_{0,\text{T}}}{E_{x,90}} = \varepsilon_{90,\text{T}} + \frac{\sigma_{90,\text{T}}}{E_{x,0}} \tag{8}$$

$$t_0\sigma_{0,\text{T}} + t_{90}\sigma_{90,\text{T}} = 0 \tag{9}$$

Since $\varepsilon_{0,\text{T}}$ and $\varepsilon_{90,\text{T}}$ are known from Equations 6 and 7, both $\sigma_{0,\text{T}}$ and $\sigma_{90,\text{T}}$ can be calculated from the set of Equations 8 and 9 and in a next step the overall transverse strain ε_{T} can be determined.

From the overall transverse strain ε_{T} and the longitudinal strain of the 0°-ply $\varepsilon_{0,\text{L}}$, the Poisson's ratio can be calculated:

$$\upsilon_{xy} = -\frac{\varepsilon_{\text{T}}}{\varepsilon_{0,\text{L}}} \tag{10}$$

Since all formulas contain the crack spacing 2s and thus the crack density as a parameter, a closed form solution for the overall Poisson's ratio is obtained as a function of crack density. The equations only contain basic material parameters in the undamaged state, which is an advantage as compared to the model of Nuismer and Tan [8, 9] and the continuum damage model of Talreja [7, 10].

4.2. Comparison of experiment and theory

To check the validity of the model, experiment and theory were compared for the cross-ply lay-ups tested during this work. Most of the necessary material parameters are given in Table I. G_t was determined by performing a tensile test on a $[45_8]$ laminate and was calculated to be 4.7 GPa.

Fig. 4 shows the model predictions and the obtained experimental results for the three lay-ups under consideration. The model curves follow the general trend in the degradation behaviour very well for all lay-ups under study. The effect of the $0^\circ/90^\circ$ ply thickness ratio is predicted by the model and the quantitative agreement between model and experiment is reasonably good.

The model developed here thus offers a good description of the Poisson's ratio degradation in a variety of cross-ply lay-ups. Compared to the models presented in the literature, it offers certain distinct advantages. The first of these advantages is that it only uses properties of undamaged laminates that can be determined by performing simple tensile tests. This is in contrast to the model of Nuismer and Tan [8, 9] which uses stiffness properties that are difficult to determine.

The second advantage is that the model presented here offers a better quantitative prediction of the data. Both the model of Smith and Wood [6] and the model of Talreja [7, 10] exhibited significant discrepancies between the experimental and the theoretical results. The model proposed in this paper gives a good agreement over the whole range of matrix crack densities.

5. Conclusions

A systematic study was made of the influence of transverse matrix cracking on the longitudinal Young's modulus and the Poisson's ratio of a variety of cross-ply lay-ups. Large reductions in Poisson's ratio were observed as the matrix crack density increased, while little or no reduction in Young's modulus was noticeable. An explanation for this behaviour was given based on shear lag theory and the known longitudinal and transverse strain state in the different plies.

The Poisson's ratio is thus much more sensitive to the phenomenon of transverse matrix cracking. This observation could be used in e.g. a continuous damage monitoring system for composite materials based on the degradation of mechanical properties. An advantage of this type of system would be its simple construction since it relies only on well developed strain monitoring techniques. However, it is up to now not known whether the same sensitivity will be obtained in other lay-ups (e.g. quasi-isotropic) or when other damage phenomena (e.g. delamination) are present. In an attempt to predict the Poisson's ratio degradation, a model was developed which is based mainly on the shear lag theory. A comparison of the experimental and the theoretical results showed that a good qualitative and quantitative agreement was obtained for all cross-ply lay-ups under consideration.

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